MSW-BANGLE FUNCTIONS AND GENERIC BASIS FOR SURFACE CLUSTER ALGEBRAS

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This is a report on a joint project with Daniel Labardini-Fragoso and Jon Wilson.

Let $\mathbf{\Sigma} := (\Sigma, \mathbb{M}, \mathbb{P})$ be a marked surface in the sense of Fomin-Shapiro-Thurston with $\mathbb{M} \subset \partial\Sigma$ the marked boundary points, and $\mathbb{P} \subset (\Sigma \setminus \partial\Sigma)$ the punctures. We suppose $|\mathbb{M}| \geq 2$ and in particular $\partial\Sigma \neq \emptyset$. In this situation the cluster algebra $\mathcal{A}(\mathbf{\Sigma})$ with trivial coefficients is locally acyclic, and the generic CC-functions form a basis.

We show, that in this situation the just mentioned *generic basis coincides with the combinatorially defined MSW-bangle functions.* In particular, the bangle functions form a basis of $\mathcal{A}(\mathbf{\Sigma})$.

This was previously only known for $\mathbb{P} = \emptyset$. Important ingredients of our proof are the following considerations:

(1) $\mathbf{\Sigma}$ admits a tagged triangulation T of signature 0. The corresponding nondegenerate Jacobian algebra $A(T) := \mathcal{P}_{\mathbb{C}}(Q(T), W(T))$ is skewed-gentle.

(2) The description of homomorphisms between finite dimensional indecomposable representations of skewed-gentle algebras from [Ge99] is completed. This allows us, in view of (1), to conclude that for *each* tagged triangulation T' there is an isomorphism of partial KRS-monoids

$$\pi_{T'}\colon\ \mathrm{DecIrr}^{\tau}(A(T')) \to \mathrm{Lam}(\mathbf{\Sigma}),$$

which intertwines generic g-vectors and shear coordinates with respect to T'.

(3) For each primitive loop λ on $\mathbf{\Sigma}$ there exists a (tagged) triangulation T'', where we can verify, with the help of Haupt's formula, that the generic CC-function for $\pi_{T''}^{-1}(\lambda)$ is the same as the MSW-function for λ with respect to T''.

(4) Since generic CC-functions and MSW-functions transform in the same way under flips of triangulations, our claim follows by combining (2) and (3).